A Multi-Resolution Method for CT Image Reconstruction

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Abstract
CT technology is a new technology which combines ray optoelectronics, informatics, microelectronics and other disciplines. Because of its advanced non-destructive testing technology, it is widely used in many fields such as medicine, biology and so on. Image reconstruction is to reconstruct its own image by using the projection of the cross-section of the object. It does not need to dissect the object. Therefore, image reconstruction in the medical field, as an advanced detection technology, can more accurately detect the internal organs of the human body. In this paper, a multi-resolution reconstruction method for CT images is proposed. The input image is first convoluted by Fourier transform to eliminate the periodic overlap effect of cyclic convolution. The image pixel matrix coefficients are transformed in frequency domain and denoised by filters, so that the low-frequency part of the coefficient matrix is retained and the high-frequency part is removed, because most of the noise will appear in the high-frequency part. The fast Fourier transform and its inverse Fourier transform algorithm are used, and the smoothness factor is added in the filtering process. For each direction of projection, the back projection is carried out to obtain the reconstructed CT image. The advantage of this method is that the final reconstructed image has a significant improvement in spatial resolution, and the background noise of the image is also reduced to a large extent. The experimental results show that this method can better preserve the image details, ensure the denoising effect of the reconstructed image, improve the accuracy of the reconstructed image, and obtain high-quality images.

Key words: CT Images, Multi-Resolution Reconstruction, Projection Transformation of Parallel Beams.

Un Método de Resolución Múltiple para la Reconstrucción de Imágenes por TC

Resumen
La tecnología CT es una nueva tecnología que combina la optoelectrónica de rayos, la informática, la microelectrónica y otras disciplinas. Debido a su avanzada tecnología de pruebas no destructivas, se usa ampliamente en muchos campos, como la medicina, la biología, etc. La reconstrucción de la imagen consiste en reconstruir su propia imagen mediante la proyección de la sección transversal del objeto. No necesita disecionar el objeto. Por lo tanto, la reconstrucción de imágenes en el campo médico, como tecnología de detección avanzada, puede detectar con mayor precisión los órganos internos del cuerpo humano. En este documento, se propone un método de reconstrucción de resolución múltiple para imágenes de TC. La imagen de entrada primero se enrosca mediante la transformada de Fourier para eliminar el efecto de superposición periódica de la convolución cíclica. Los coeficientes de la matriz de píxeles de la imagen se transforman en el dominio de la frecuencia y se eliminan mediante filtros, de modo que la parte de baja frecuencia de la matriz de coeficientes se mantiene y la parte de alta frecuencia se elimina, porque la mayor parte del ruido aparecerá en la parte de alta frecuencia. Se utilizan la transformada rápida de Fourier y su algoritmo inverso de transformada de Fourier, y el factor de suavidad se agrega en el proceso de filtrado. Para cada dirección de proyección, la proyección hacia atrás se lleva a cabo para obtener la imagen de CT reconstruida. La ventaja de este método es que la imagen reconstruida final tiene una mejora significativa en la resolución espacial, y el ruido de fondo de
la imagen también se reduce en gran medida. Los resultados experimentales muestran que este método puede preservar mejor los detalles de la imagen, garantizar el efecto de eliminación de ruido de la imagen reconstruida, mejorar la precisión de la imagen reconstruida y obtener imágenes de alta calidad.

**Palabras clave:** Imágenes de TC, Reconstrucción con Resolución Múltiple, Transformación de Proyección de Haces Paralelos

1. Introduction

Computerized tomography (CT) is an imaging technique that uses X-ray to illuminate objects from multiple angles to obtain projection data, and then uses specific mathematical methods to reconstruct the tomographic images of objects. Since its invention, the technology has been widely used in many disciplines, such as industrial testing, medicine, military, public safety, biology and so on, with its advantages of high fraction, high sensitivity and non-destructive testing. Region. Among them, the role in the field of medical diagnosis is particularly prominent [1]. CT is also of great value in the diagnosis of five senses and some diseases. For the diagnosis of some diseases, image enhancement scanning is usually used to determine whether there is no tumor mass or lymph node enlargement in mediastinum and porta, whether there is tumor or obstruction or obstruction in supporting trachea, primary or metastatic mediastinal tumors, Barba tuberculosis, cardiac tumors, cardiac tumors, etc. Diagnosis is very helpful. If high quality image can be reconstructed, it depends on the reconstruction algorithm used. Image reconstruction is the process of reconstructing cross-section by projecting the cross-section of the object to the plane and according to the function obtained by the projection. According to the different acquisition methods and reconstruction rationale of raw data, it can be divided into transmission tomography reconstruction, emission tomography reconstruction and reflection tomography reconstruction[2]. CT technology often uses high radiation dose to obtain high-quality tomographic images with higher segmentation rate and better visual effect. However, high-dose X-ray irradiation of human body may cause many kinds of disease symptoms, such as symptoms, gene mutations, leukemia, etc. Therefore, X-ray dose in CT image reconstruction has attracted wide attention. The dose of low-dose X-rays has attracted wide attention. Because the low dose radiation scanning will bring more noise and the data is incomplete, it is important to study how to reconstruct high quality CT images using the noise images and incomplete projection data [3].

The application of image reconstruction technology in X-ray medicine is a major breakthrough, which is of great significance to the diagnosis of visceral diseases. Computerized Tomography technology was invented by American scientist A. M. Cormark and British scientist G. N. Hounsfield from 1950s to 1970s through a series of research and experiments in nuclear physics, nuclear medicine and other fields. Since the first generation of CT equipment for clinical application came out in 1971, with the rapid development of electronic technology, CT technology has been constantly improved. New types of equipment such as spiral CT machine and electron beam scanner have gradually been widely used by medical institutions to obtain high-quality images [4]. Different reconstruction methods will affect the speed and quality of reconstructed images, so efficient and fast reconstruction methods have always been CT. The traditional iterative algorithm for CT image reconstruction has the disadvantages of large computational complexity and slow reconstruction speed [5]. CT image reconstruction methods are mainly divided into analytical method and algebraic method. The analytical method relies on Radon transform and its inversion formula, such as filtering back projection (FBP) reconstruction method. Analytical method has low computational complexity and fast reconstruction speed, but only when the projection data is complete, that is, the projection data on all straight lines passing through the object on the plane need to be known, and the data error is small, the better reconstruction results can be obtained by analytic method [6].

Firstly, this paper introduces the principle of CT tomography, including physical and mathematical principles. In image transformation, the essence of transformation is to calculate the projection of image matrix in a certain direction. Because the projection of an object can be obtained by illuminating light, if we can get the projection of the object in different directions, we can inversely transform these projection images to reconstruct its two-dimensional structure or three-dimensional structure. Then, the principle of multi-resolution CT dimensional image reconstruction method is given. Finally, the experimental results prove the effectiveness of the proposed algorithm.

2. Principle of CT Tomography

The process of CT imaging can be summarized as follows: scan the object to be detected with equipment and receive with detector the X-ray passing through the object. The X-ray becomes digital signal, i.e. original data after analog-to-digital conversion. Then after such pre-processing as beam hardening and relieving zero
drift, obtain more accurate projection data. Finally, after image reconstruction and processing, obtain the output display of faulted image and store it in the equipment [7] [8].

Tomography imaging technology illuminates the object with parallel X-ray from different directions and records the transmission field of every direction. The parallel X-ray generated from X-ray source lights the object. Assume that the incident light intensity is $I_0$ and that the transmission photometry received by the receiver array is $I(t, \theta)$. In the same way, turn X-ray source and X-ray receiver at an angle to $\theta$ along the center and obtain a group of projection data $I(t, \theta_2)$. The scope of $\theta$ is 0° to 180°. Make some changes at regular intervals. Then we can have the projection vector.

The projection of the image can be described as follows: the image function is $f(x, y)$ and one line that passes this line is referred to as ray. If the integral value of the function at a certain ray forms a collection, then this collection represents the projection value [9].

Make a perpendicular from the origin to the ray. This ray as an axis of new coordinates, constitutes a new coordinate system $(t, s)$. In this way, it can be seen that coordinate system $(t, s)$ is only the result of rotating the coordinate system $(x, y)$ by $\theta$ degrees and that the two of them have the following transformation relation.

$$
\begin{bmatrix}
t \\
s
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
$$

So, the integral expression of the ray is as follows:

$$P_x(t_i) = \int_{-\infty}^{\infty} f(x, y) dx dy = \int_{-\infty}^{\infty} f(t_i, s) ds$$

Tomography imaging technology is the process to reconstruct the original image $f(x, y)$ with the projection values $P_x(t)$ obtained from different ray angles $\theta$ and positions of different detectors. $f(x, y)$ reflects the density at $(x, y)$.

Statistical fluctuation usually occurs to a CT image or even the ray attenuation value, as shown by the coarsening of image grains, in another word, increased image noises. The severity of statistical fluctuation is usually inversely proportional to ray dose in the scanning. In other words, the less ray dose adopted in the scanning, the more severe the statistical fluctuation and the more image noises. Besides, image noises can be represented with the standard deviation of mean. In the CT attenuation measurement, the region of interest adopted shall be of a certain size in order to make the mean as accurate as possible. So, sometimes punctuate CT attenuation measurement is not accurate enough [10].

3. Radon Transform

In image transform, Radon transform in essence, is to calculate the projection of image array at a certain direction. If the image array is 2D, the projection at a certain direction is the line integral along its perpendicular direction.

In the rectangle coordinate system, $f(x, y)$ refers to the point at line $I$ and $P$ is the distance from the coordinate origin to the line $I$ and it means the angle along the normal direction of line $I$. So, the equation of straight line can be represented as follows:

$$x \cos \theta + y \sin \theta = P$$

The formula of Radon transform at line $I$ is:

$$\text{Radon}_I(P, \theta) = \int_{-\infty}^{\infty} f(x, y) dl$$

Delta function is a generalized function with no specific definition. This function takes 0 as its value at the non-zero points and its integral of the entire domain of definition is 1. Please find the simplest Delta function here for your understanding.
\[
\delta(x) = \begin{cases} 
0, & x \neq 0 \\
1, & x = 0 
\end{cases}
\] (5)

In combination with the equation of straight line, Delta function can be represented as follows:

\[
\delta(P - x \cos \theta - y \sin \theta) = \begin{cases} 
0, & P - x \cos \theta - y \sin \theta \neq 0 \\
1, & P - x \cos \theta - y \sin \theta = 0 
\end{cases}
\] (6)

Namely that the point \((x, y)\) in line \(l\) meets \(\delta(x) = 1\) while other points not in line \(l\) meet \(\delta(x) = 0\).

Radon transform can be applied in X-ray CT scan and protein complex and it is also the solution to hyperbolic partial differential equation. What is most frequently used in Radon transform is its inverse transform. As the projection of an object can be obtained by light, if we can obtain the projection of this object in different directions, we can perform Radon inversion to these projected images so as to reconstruct its 2D or 3D structure [11] [12].

![Figure 1. Radon transform of 90 angles of brain](image1)

![Figure 2. Radon transform of brain image](image2)
4. Algorithm of CT Image Reconstruction

To reconstruct the image of an object from projection is to reconstruct the cross sections one by one. In this way, the reconstruction of image is actually the reconstruction of image. And the reconstruction problem of image can be mathematically described as follows.

Assume that \( g(x, y) \) represents an unknown function and the straight line through \( g(x, y) \) is called light. The integral along light \( g(x, y) \) is called light integral. A group of light integrals along the same direction constitute a projection.

Assume that the projection direction is \( \theta \). Rotate the coordinate \((x, y)\) by \( \theta \) degrees (counter-clockwise) and form the coordinate system \((t, s)\). \( g(x, y) \) is \( g(t, s) \) in the coordinate system of \((t, s)\).

Obviously,

\[
P_\theta(t) = \int_{-\infty}^{\infty} g(t, s) ds
\]

Make \( S_\theta(w) \) as the Fourier transform of \( P_\theta(t) \). Then

\[
S_\theta(w) = \int_{-\infty}^{\infty} P_\theta(t) \exp(-j2\pi wt) dt = \int_{-\infty}^{\infty} g(t, s) \exp(-j2\pi wt) ds dt
\]

Transform the above formula into the coordinate system \((x, y)\) and get

\[
S_\theta(w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \exp[-j2\pi(x \cos \theta + y \sin \theta)] | \int | dx dy
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \exp[-j2\pi(ux + vy)] dx dy
\]

Here,

\[
\begin{cases}
  u = \omega \cos \theta \\
  v = \omega \sin \theta
\end{cases}
\]

If \( G(x, y) \) is made as the Fourier transform of \( g(x, y) \), it can be learnt from Formula (10) that

\[
S_\theta(\omega) = G(u, v) = G(\omega, \theta)
\]

If the Fourier transform of \( g(x, y) \) is the polar coordinate representation of \( G(u, v) \), it means that \( g(x, y) \) is the projection \( P_\theta(t) \) at direction \( \theta \).

Fourier transform \( S_\theta(\omega) \) is equal to the value of \( G(u, v) \) in the straight line which forms the angle \( \theta \) with \( u \) axis. So, it can be seen that in the entire \((u, v)\) plane, \( G(u, v) \) can be obtained by using the projections of every direction and \( g(x, y) \) can also be found by seeking the inverse Fourier transform of \( G(u, v) \).

\[
g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(u, v) \exp[-j2\pi(ux + vy)] du dv
\]

Transform into the polar coordinates,

\[
\begin{cases}
  u = \omega \cos \theta \\
  v = \omega \sin \theta
\end{cases}
\]

Obtain

\[
g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(\omega, \theta) \exp[j2\pi \omega(x \cos \theta + y \sin \theta)] d\omega d\theta
\]

It is derived that
\[ g(x, y) = \int_0^\pi \left[ \int_{-\infty}^{\infty} S_\phi(\omega) |\omega| \exp(j2\pi\omega t) d\omega \right] d\theta \]  

Here,  
\[ t = x \cos \theta + y \sin \theta \]  

If  
\[ Q_\phi(t) = \int_{-\infty}^{\infty} S_\phi(\omega) |\omega| \exp(j2\pi\omega t) d\omega \]  

then,  
\[ g(x, y) = \int_0^\pi Q_\phi(x \cos \theta + y \sin \theta) d\theta \]  

The right side of Formula (18) is the inverse Fourier transform of the product of two spectrum functions:  
\[ S_\phi(w) \text{ and } H(\omega) \].  
\[ S_\phi(w) \] is the Fourier transform of projection  
\[ P_t(\omega) \]. If the inverse Fourier transform of  
\[ H(\omega) \] is  
\[ h(t) \], then according to convolution theorem.  

\[ Q_\phi(t) = \int_{-\infty}^{\infty} P_\phi(\tau) h(t-\tau) d\tau \]  

or  
\[ Q_\phi(t) = P_\phi(t)^* h(t) \]  

Here,  
\[ h(t) = \int_{-\infty}^{\infty} |\omega| \exp(j2\pi\omega t) d\omega \]  

When the spectrum of the image is bandwidth limited, the above formula becomes:  
\[ h(t) = \int_{-\omega_b}^{\omega_b} |\omega| \exp(j2\pi\omega t) d\omega \]  

As the image and its spectrum are discretely sampled, assume that the interval is  
\[ \tau \], then according to the sampling theorem,  
\[ \omega_b = 1/2\tau \]. In order to perform mathematical processing, we only need to know the value of  
\[ h(t) \] in limited bandwidth. In this way, we can find  

\[ h(n\tau) = \begin{cases} 
1/(4\tau^2) & n = 0 \\
-1/n^2\pi^2\tau^2 & 0 < n < N 
\end{cases} \]  

Here,  
\[ n \] is integer.  
The discrete form of Formula (23) is as follows:  
\[ Q_\phi(n\tau) = \tau \sum_{m=-\infty}^{\infty} P_\phi(m\tau) |\tau| (n-m) \tau \]  

In order to obtain  
\[ Q_\phi(n\tau) \] , calculate the convolution with Fourier transform. In order to remove the periodic overlap effect of cyclic convolution, take 2N points in  
\[ h(n\tau) \] and take non-zero as the value of  
\[ P_\phi(m\tau) \] so that there are (2N-1) elements. Then,  
\[ P_\phi(t) \] has no overlap in N sampling points. If the fast Fourier transform (FFT) algorithm with 2 as basis is adopted, both  
\[ P_\phi(m\tau) \] and  
\[ h(n\tau) \] will have 0 to (2N-1) elements and (2N-1) is the integer power of 2 bigger than or equal to 2N-l. The process to calculate v can be written as follows:
Here, FFT and IFFT represent fast Fourier transform and inverse fast Fourier transform respectively. Smooth window refers to the smooth factor added in the filtering process. For the projection of every $\theta$ direction, calculate $g(x, y)$ according to Formula (12) after obtaining $Q_\theta(n\tau)$. The reconstruction steps are as follows.

Step 1: Convolution, also known as filtering. Calculate $Q_\theta(n\tau)$ in every $\theta$ direction based on Formula (15).

Step 2: Back projection. The approximate value $\hat{g}(x, y)$ of $g(x, y)$ can be calculated according to the approximate form of Formula (18).

\[
g(x, y) = \frac{\pi}{M} \sum_{i=1}^{M} Q_{\theta_i}(x \cos \theta_i + y \sin \theta_i)
\]

$M$ is the number of projections and $\theta_i$ is the projection angle. They are uniformly districted in the range of $0 \sim \pi$.

5. Experiments and Analysis

In order to ensure the accuracy of data processing, the experiment first preprocessing is to compensate the acquired data in a variety of ways. Then, convolution is used to select different filters according to different image diagnostic requirements. Then, back projection reconstruction is based on the attenuated X signal obtained to obtain the density information of the scanning object in all directions and convert it into the original data and transmit it to the image reading system. Medium. Initially input low-resolution CT brain images are reconstructed by Radon transform and the method presented in this paper. The results are shown in Fig. 3 and Fig. 4 below.

(a) Original brain image  
(b) Projection transformation of parallel beams

**Figure 3.** Projection transformation of parallel beams
From the above figures, we can see that the reconstruction effect of this algorithm is better. This is mainly because the algorithm has better stability. In addition, it performs better denoising for CT images in advance, which makes the proposed algorithm reduce image artifacts in the process of updating images, and at the same time simplifies the calculation process of low rank matrix. In order to reduce the complexity of the algorithm, Fourier transform and low rank matrix are used to simplify the calculation process and reduce the operation time of the algorithm. Experiments show that the proposed method has good denoising effect, strong detail preservation ability, improved the convergence rate of the algorithm, and reconstructed high-quality CT images.

6. Conclusions

CT images can clearly show the structural characteristics and pathological changes of human organs, and provide scientific basis for the correct diagnosis of diseases in central nervous system, heart, head and neck. In medical image processing, image reconstruction is often achieved by projecting multiple X-rays on a certain section to obtain the structural graphics of the section. In this paper, the multi-resolution algorithm is used to post-process the image in CT scanner to improve the final spatial resolution of the image. In the experiment, the brain resolution image of CT was used as input image, and the output high resolution image was reconstructed with multi-resolution. The experimental results show that the proposed algorithm can help improve the image quality in an economical and feasible way without upgrading the hardware of computed tomography scanner.

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References


